Engineering UTC
Northern
Lincolnshire

Mathematics Faculty Approved Methods Booklet

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RATIONALLE

Use

This booklet was created to be used by the following:

1. Students:

   Students can use this booklet to remind themselves, using the worked examples, of how to answer certain types of questions when their teacher is not accessible (such as revising at home) or when in a lesson with a teacher who is not a maths specialist.

2. Teachers:

   Teachers can use this booklet to refer back to how students should be approaching certain topics in their lessons and to support with methods they may not be familiar with

3. Parents/guardians:

   Parents and guardians can use this booklet to support their children with any work they may be trying to complete outside of school such as independent revision. The biggest thing that puts parents off supporting their children with maths is that it's 'not done the way I did it', hopefully this book can help introduce the current methods we use in school.

Methods

There are two types of methods included in this booklet. Some types of questions have both but some only have one. The reason is as follows:

1. Mastery method

   This is the most effective and most mathematically coherent method for answering the question that we will be teaching at the UTC.

2. Procedural method

   This method is not as mathematically rigorous but is widely taught across other schools. Because we are aware that by enrolling students is year 9 they may already have established methods that work for them we have included these methods as an acceptable alternative to the mastery method.

MULTIPLICATION

Mastery method – ‘Traditional’ or ‘Column method’	Method
352 × 26 $\begin{array}{r} 3\ 5\ 2 \\ \times \quad 2\ 6 \\ \hline \\ \\ \hline \\ \hline \end{array}$	Set up the question with the longest number on top. Leave one line of working per digit of the second number (2 digits, 2 lines) followed by an answer line.
$\begin{array}{r} 3\ 5\ 2 \\ \times \quad 2\ 6 \\ \hline {}^{1}2 \\ \\ \hline \\ \hline \end{array}$	Multiply each integer in the top number by the first integer in the second row. Where your answer is bigger than 10, ‘carry’ the tens and write in the digits. In this example, 2 x 6 is 12 so carry the 1 and write in the two
$\begin{array}{r} 3\ 5\ 2 \\ \times \quad 2\ 6 \\ \hline 2\ 1^{3}\ 1^{1}\ 2 \\ 0 \\ \hline \\ \hline \end{array}$	When multiplying by the second digit (in this case 2) remember to put a 0 at the start of the row as you are multiplying by 10’s. Then do the same thing you did for the first row.
$\begin{array}{r} 3\ 5\ 2 \\ \times \quad 2\ 6 \\ \hline 2\ 1^{3}\ 1^{1}\ 2 \\ 7^{1}\ 0\ 4\ 0 \\ \hline 9\ 1\ 5\ 2 \\ \hline \end{array}$	Once you have completed the two middle ‘working’ rows, add the digits up like you would any other column addition to get your final answer

DIVISION	
Mastery method – Short division or ‘bus stop’ method	Method
711 ÷ 3 $3\overline{)7\;1\;1}$	Set the question up by placing the dividend (the number being divided) inside the ‘bus stop’ and the divisor (the number you are dividing by) on the outside.
$\begin{array}{l} \quad 2 \\ 3\overline{)7\;{}^{1}1\;1} \end{array}$	The next step is to see how many times 3 goes in to the first digit, 7 in this case. 3 goes in to 7 twice so the 2 goes on the top and the spare one which is left over is carried to the next digit.
$\begin{array}{l} \quad 2\;\;3 \\ 3\overline{)7\;{}^{1}1\;{}^{2}1} \end{array}$	Continue to do this with each digit in order. So to the left you can see that 3 goes into 11, 3 times with 2 left over so the 3 goes on the top and the 2 is carried over to the next digit.
$\begin{array}{l} \quad 2\;\;3\;\;7 \\ 3\overline{)7\;{}^{1}1\;{}^{2}1} \end{array}$	Finally we check how many times 3 goes in to 21. This is 7 exactly so the 7 goes on the top and there is nothing to carry.

SIMPLIFY FRACTIONS

Mastery method – Divide numerator and denominator	Method
Simplify $\frac{6}{18}$ $$\frac{6}{18} \xrightarrow{\div 6} \frac{1}{3}$$ TIP: You don’t have to divide by the highest number first if you aren’t sure what it is. In the example above you could divide by 2 then divide your answer by 3 and still get the same answer.	The objective of simplifying a fraction is to have a fraction with the same value but with the smallest possible integers (whole numbers) on the top and the bottom. We do this by dividing both the numerator (top) and denominator (bottom) by the highest number that goes in to them both. In this case 6.

SIMPLIFY RATIOS

Mastery method – Divide both sides of the ratio	Method
Simplify 6:18 $$\div 6 \left(\begin{array}{c} 6 : 18 \\ \downarrow \\ 1 : 3 \end{array} \right) \div 6$$ TIP: You don’t have to divide by the highest number first if you aren’t sure what it is. In the example above you could divide by 2 then divide your answer by 3 and still get the same answer.	The objective of simplifying a ratio is to have the smallest possible integers (whole numbers) on both sides of the ratio. We do this by dividing both sides by the highest common factor (the biggest number that goes into both of the parts). In this case 6.

<table>
<tr><td colspan="2">ADD AND SUBTRACT FRACTIONS</td></tr>
<tr><td>Mastery method – Change to the same denominator</td><td>Method</td></tr>
<tr><td>Calculate $\frac{1}{3}+\frac{3}{7}$

$\frac{1}{3} + \frac{3}{7}$

$\frac{1}{3} \xrightarrow{\times 7} \frac{7}{21}$ $\frac{3}{7} \xrightarrow{\times 3} \frac{9}{21}$

$\frac{7}{21} + \frac{9}{21}$

$\frac{7}{21} + \frac{9}{21} = \frac{16}{21}$</td><td>To answer this question we first have to change the denominator into the same number. We do this by finding the smallest number that both of the denominators go into (it's lowest common multiple).

Once we have changed the denominators we have to multiply the numerator by the same number we used to chance the denominator.

Once complete we can just add the numerators.</td></tr>
<tr><td colspan="2">SOLVE EQUATIONS</td></tr>
<tr><td>Mastery method – Balance method</td><td>Method</td></tr>
<tr><td>Solve $3x + 2 = 17$

$3x + 2 = 17$
$(-2) \quad (-2)$

$3x = 15$
$(\div 3) \quad (\div 3)$

$x = 5$</td><td>Our mastery method for this question involves showing what you're doing to each side during each part of the process.

We want to remove the thing least connected to the x first. In this case 2. Because it's +2 we need to -2 to get rid of it. Because we're using the balance method we have to do the same to both sides.

We continue this until the question is solved.</td></tr>
</table>

SHARING IN A RATIO	
Mastery method – Scaling a ratio	Method
Share £210 in the ratio 4:3	
Total 4 : 3 7	Set up the question by writing out the given ratio and the total parts as shown to the right.
Total 4 : 3 7 210	Because we are sharing the total, we put the total amount to be shared under the total amount of parts.
Total 4 : 3 $7 \xrightarrow{\times 30} 210$	We can then calculate the 'multiplier' that is needed to make 7 parts into the 210 we were asked to share. In this case it's 30.
Total $4 : 3 \xrightarrow{\times 30} 120 : 90$ $7 \xrightarrow{\times 30} 210$	Because the multiplier for the total is 30 the multiplier for each separate part is also 30. So we can multiply each part to get our different amounts after sharing.

SHARING IN A RATIO

Procedural method – 'Add, divide, multiply'	Method
Share £210 in the ratio 4:3 ① $4 + 3 = 7$	Step one involves adding both parts of the ratio together to get the total parts.
① $4 + 3 = 7$ ② $210 \div 7 = 30$	Step two is dividing the total we want to share by the total amount of parts that we found during step 1. This finds the value of each part.
① $4 + 3 = 7$ ② $210 \div 7 = 30$ ③ $4 \times 30 = 120$ $3 \times 30 = 90$	Step 3 is multiplying the value of one part by the size of each part. In this case that is 4 and 3.

MORE COMPLEX RATIO

Mastery method – Scaling a ratio	Method
Bill and Ted share some money in the ratio 7:11 Ted gets £32 more than Bill. How much was shared?	
$\begin{array}{ccccc} B & : & T & \text{Diff} & \text{Total} \\ 7 & : & 11 & 4 & 18 \end{array}$	Set the question up by laying out all of the information we have. Bill's share, Ted's share, the difference and the total. The difference is calculated because we are given the difference in the question.
$\begin{array}{ccccc} B & : & T & \text{Diff} & \text{Total} \\ 7 & : & 11 & 4 & 18 \\ & & & \downarrow \times 8 & \\ & & & 32 & \end{array}$	Because £32 is the difference it goes under difference and we work out the multiplier to get from the difference in the ratios to the difference in the amounts shared.
$\begin{array}{ccccc} B & : & T & \text{Diff} & \text{Total} \\ 7 & : & 11 & 4 & 18 \\ & & & \downarrow \times 8 & \downarrow \times 8 \\ & & & 32 & 144 \end{array}$	Because we are finding the total that was shared we use this multiplier on the total amount of the ratio. NOTE: You could also multiply both Bill and Ted's shares and add them together to get the same total.

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MORE COMPLEX RATIO

Procedural method – 'Subtract, divide, multiply'	Method
Bill and Ted share some money in the ratio 7:11 Ted gets £32 more than Bill. How much was shared? 1 $11-7=4$	Step one is to work out the difference in the ratios because we are given the difference in the original amounts.
1 $11-7=4$ 2 $32\div4=8$	Step two is to divide the difference in amounts by the difference in parts to find what one part is worth.
1 $11-7=4$ 2 $32\div4=8$ 3 $7+11=18$ $18\times8=144$	Step 3 is to find the total amount of parts and multiply it by the value of each individual part that we found in step 2.

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EXPANDING A SINGLE BRACKET	
Mastery method	Method
Expand $7(x+4)$	When we say ‘expand’ what we mean is multiply the part on the outside by both of the parts on the inside. We can track what we have already multiplied using lines but once you are comfortable with these questions you might choose not to use them.
$7(x+4)$ $7x$	7 multiplied by x gives us the algebraic term 7x as the first part of our expansion.
$7(x+4)$ $7x+28$	For the second step 7 is multiplied by 4, giving us the answer 28 as our next term.

EXPANDING AND SIMPLIFYING A POLYNOMIAL	
Mastery method – Splitting the brackets	Method
Expand and simplify $(x+5)(x+9)$ $x(x+9)+5(x+9)$ $x(x+9)+5(x+9)$ x^2+9x. $x(x+9)+5(x+9)$ $x^2+9x+5x+45$ $x^2+14x+45$	This questions essentially means we need x and 5 lots of x and 9. Using our mastery approach we would split the first expression (bracket) to create two separate single bracket expansions of the second expression that we would expand using the skills on the previous page. Following that we would collect the two terms in the middle to simplify our final answer.

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EXPANDING AND SIMPLIFYING A POLYNOMIAL

Procedural method – Grid method	Method

Expand and simplify $(x+5)(x+9)$

This questions essentially means we need x and 5 lots of x and 9.

	x	$+5$
x		
$+9$		

We would set the question up using a grid with a column for each term in the first bracket and a row for each term in the second bracket.

	x	$+5$
x	x^2	$5x$
$+9$	$9x$	45

We would then multiply the columns and rows together to get 4 terms.

	x	$+5$
x	x^2	$5x$
$+9$	$9x$	45

+

$$x^2 + 14x + 45$$

Finally we would then collect all of the like terms to simplify the expression.

FACTORISING QUADRATICS	
Mastery method – Splitting the coefficient of x	Method
Factorise $3x^2 + 13x + 4$ $AC = 3 \times 4 = 12$ $\times$ 12 $+$ 13 12 and 1 $3x^2 + 12x + x + 4$ $3x(x+4) + 1(x+4)$ $(3x+1)(x+4)$	For our mastery method we start with the more complex examples of factorising as these are applicable to ALL questions. To factorise this quadratic we need to find the AC value (the coefficient of x^2 multiplied by the constant) which in this case is 12. We then find 2 numbers that multiply to make the AC value and add to make the coefficient of x, 13 in this case. We then split the middle coefficient into the two numbers we have chosen and then factorise it as two separate single bracket factorisations. This is a complex process and we don't expect you to be able to do it from this help sheet this is purely to give you a nudge in the right direction if you're stuck and to make you aware of the method we prefer to use.

CALCULATIONS WITH TIME	
Mastery method – Using minutes and converting	Method
Harry and Lloyd travel 150 miles in 2 hours 20 mins. What is their average speed to the nearest mph. $S = \frac{D}{T}$ 2 hours 20 mins = 140 mins $\frac{150}{140} \times 60 = 64.28$ mph $\Downarrow$ 64 mph	Using our mastery method we would convert the hours and minutes to minutes, perform the calculation needed then multiply by 60 to convert back to miles per hour.

CALCULATIONS WITH TIME

Procedural method – Known minute to decimals	Method

Harry and Lloyd travel 150 miles in 2 hours 15 mins. What is their average speed to the nearest mph.

Minutes	Decimal
60	1
30	0.5
15	0.25
45	0.75
6	0.1

2 hours 15 mins = 2.25 hours

$$\frac{150}{2.25} = 66.\dot{6} \text{ mph}$$

$$\Downarrow$$

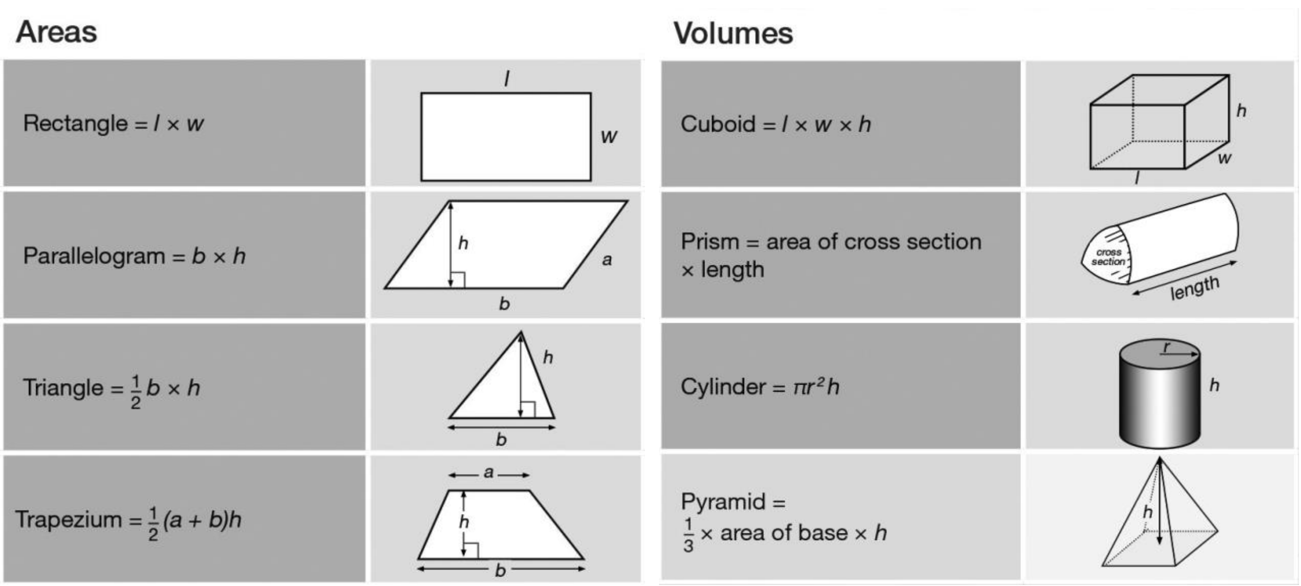
67mph

Although the mastery method works for any given time there are occasions where it is easier to convert the minutes to hours. This is when there is a clear conversion as seen in the table to the left.

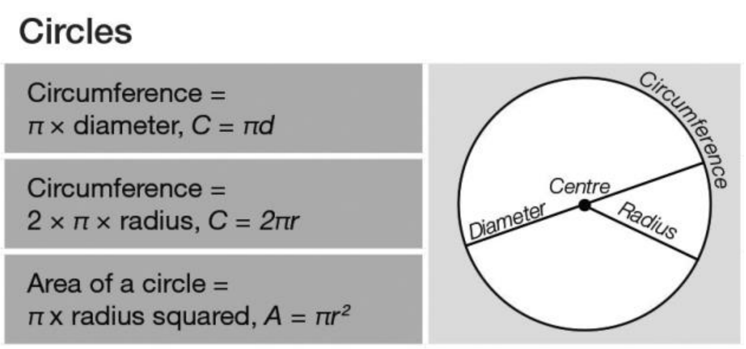
So in this question we can easily change the decimal and perform the calculation that way.

AREA CALCULATIONS

These are included not for a mastery approach but to standardise the labels used for sides in shapes. When calculating for your own purposes it's not hugely important what you call each side but the exam board have a set way of labelling shapes so it's worth being aware of what they call them.



Areas		Volumes	
Rectangle = $l \times w$		Cuboid = $l \times w \times h$	
Parallelogram = $b \times h$		Prism = area of cross section × length	
Triangle = $\frac{1}{2} b \times h$		Cylinder = $\pi r^2 h$	
Trapezium = $\frac{1}{2}(a + b)h$		Pyramid = $\frac{1}{3}$ × area of base × h	



Circles	
Circumference = π × diameter, $C = \pi d$	
Circumference = 2 × π × radius, $C = 2\pi r$	
Area of a circle = π x radius squared, $A = \pi r^2$	